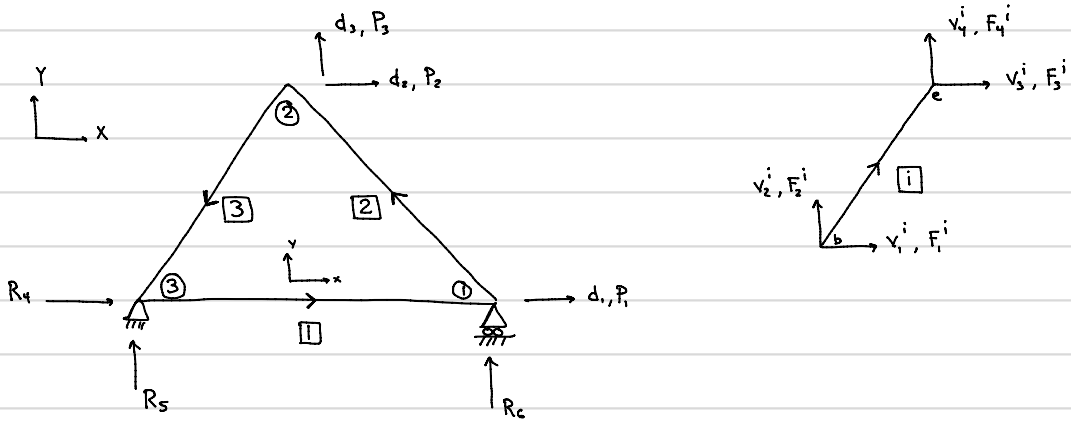
March 03, 2026
Matrix Displacement Method
2D Trusses
Part II - Assembly

# 2D Truss Assembly

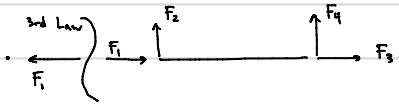




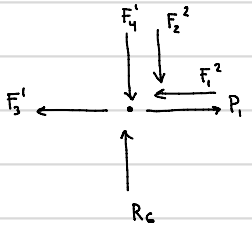


## Rigorous Assembly

### Equilibrium of Joints



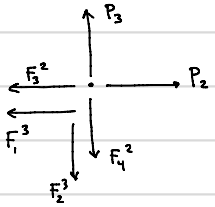

Joint ①




$$\Sigma F_x = 0 \quad P_1 - F_3^1 - F_1^2 = 0 \quad P_1 = F_3^1 + F_1^2 \ *$$

$$\Sigma F_y = 0 \quad R_6 - F_4^1 - F_2^2 = 0 \quad R_6 = F_4^1 + F_2^2$$

Joint ②




$$\Sigma F_x = 0 \quad P_2 - F_3^2 - F_1^3 = 0 \quad P_2 = F_3^2 + F_1^3 \ *$$

$$\Sigma F_y = 0 \quad P_3 - F_4^2 - F_2^3 = 0 \quad P_3 = F_4^2 + F_2^3 \ *$$

$$P_1 = F_3^1 + F_1^2$$

$$P_2 = F_3^2 + F_1^3$$

$$P_3 = F_4^2 + F_2^3$$

$$\{F\} = [K]\{v\} \qquad F_j^i = K_{j1}^i v_1^i + K_{j2}^i v_2^i + K_{j3}^i v_3^i + K_{j4}^i v_4^i$$

$$P_1 = \left(K_{31}^1 v_1^1 + K_{32}^1 v_2^1 + K_{33}^1 v_3^1 + K_{34}^1 v_4^1\right) + \left(K_{11}^2 v_1^2 + K_{12}^2 v_2^2 + K_{13}^2 v_3^2 + K_{14}^2 v_4^2\right)$$

$$P_2 = \left(K_{31}^2 v_1^2 + K_{32}^2 v_2^2 + K_{33}^2 v_3^2 + K_{34}^2 v_4^2\right) + \left(K_{11}^3 v_1^3 + K_{12}^3 v_2^3 + K_{13}^3 v_3^3 + K_{14}^3 v_4^3\right)$$

$$P_3 = \left(K_{41}^2 v_1^2 + K_{42}^2 v_2^2 + K_{43}^2 v_3^2 + K_{44}^2 v_4^2\right) + \left(K_{21}^3 v_1^3 + K_{22}^3 v_2^3 + K_{23}^3 v_3^3 + K_{24}^3 v_4^3\right)$$

$$P_1 = \left(K^1_{31} V^1_1 + K^1_{32} V^1_2 + K^1_{33} V^1_3 + K^1_{34} V^1_4\right) + \left(K^2_{11} V^2_1 + K^2_{12} V^2_2 + K^2_{13} V^2_3 + K^2_{14} V^2_4\right)$$

$$P_2 = \left(K^2_{31} V^2_1 + K^2_{32} V^2_2 + K^2_{33} V^2_3 + K^2_{34} V^2_4\right) + \left(K^3_{11} V^3_1 + K^3_{12} V^3_2 + K^3_{13} V^3_3 + K^3_{14} V^3_4\right)$$

$$P_3 = \left(K^2_{41} V^2_1 + K^2_{42} V^2_2 + K^2_{43} V^2_3 + K^2_{44} V^2_4\right) + \left(K^3_{21} V^3_1 + K^3_{22} V^3_2 + K^3_{23} V^3_3 + K^3_{24} V^3_4\right)$$

$$\{P\} = [S]\{d\}$$

Compatibility

Joint ① $\quad d_1 = V^1_3 = V^2_1$

$V^1_4 = V^2_2 = 0$ support/boundary condition (B.C.)

Joint ② $\quad d_2 = V^2_3 = V^3_1$

$d_3 = V^2_4 = V^3_2$

Joint ③ $\quad V^1_1 = V^3_3 = 0$ B.C.

$V^1_2 = V^3_4 = 0$ B.C.

$$P_1 = \overbrace{K^1_{33} d_1 + K^2_{11} d_1}^{S_{11}} + \overbrace{K^2_{13} d_2}^{S_{12}} + \overbrace{K^2_{14} d_3}^{S_{13}}$$

$$P_2 = K^2_{31} d_1 + K^2_{33} d_2 + K^3_{11} d_2 + K^2_{34} d_3 + K^3_{12} d_3$$

$$P_3 = K^2_{41} d_1 + K^2_{43} d_2 + K^3_{21} d_2 + K^2_{44} d_3 + K^3_{22} d_3$$

$$\{P\} = [S]\{d\} \qquad P_1 = S_{11} d_1 + S_{12} d_2 + S_{13} d_3$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} K^1_{33} + K^2_{11} & K^2_{13} & K^2_{14} \\ K^2_{31} & K^2_{33} + K^3_{11} & K^2_{34} + K^3_{12} \\ K^2_{41} & K^2_{43} + K^3_{21} & K^2_{44} + K^3_{22} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$[K] = [T]^T [k] [T]$$

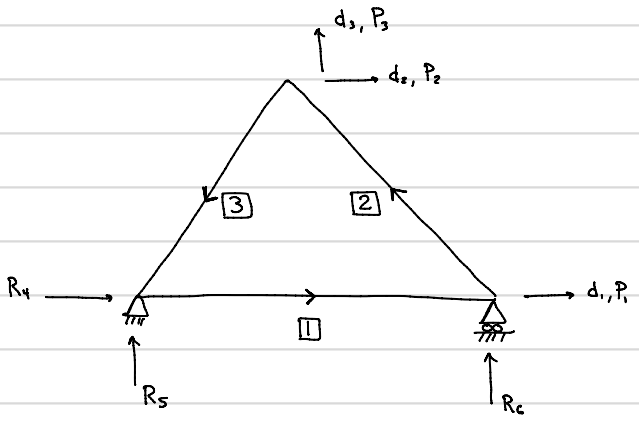
$[K]$: global $\quad [k]$: local

symmetric $\quad [M] = [M]^T$

$$[K]^T = \left([T]^T [k] [T]\right)^T$$
$$= [T]^T [k] \left([T]^T\right)^T$$
$$= [T]^T [k] [T]$$

$\therefore$ $[K]$ is symmetric

# Code # Assembly




| member # \ code # | $F_1, V_1$ | $F_2, V_2$ | $F_3, V_3$ | $F_4, V_4$ |
|---|---|---|---|---|
| 1 | 4 | 5 | 1 | 6 |
| 2 | 1 | 6 | 2 | 3 |
| 3 | 2 | 3 | 4 | 5 |

$$[k]^1 = \begin{array}{c} \\ 4 \\ 5 \\ 1 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 5 & 1 & 6 \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \end{array}$$

$$[S] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \end{array}$$

$$[k]^2 = \begin{array}{c} \\ 1 \\ 6 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 6 & 2 & 3 \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \end{array}$$

$$[S] = \begin{bmatrix} K_{33}^1 + K_{11}^2 & K_{13}^2 & K_{14}^2 \\ K_{31}^2 & K_{33}^2 + K_{11}^3 & K_{34}^2 + K_{12}^3 \\ K_{41}^2 & K_{43}^2 + K_{21}^3 & K_{44}^2 + K_{22}^3 \end{bmatrix}$$

$$[k]^3 = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \end{array}$$

Solve $\{d\} = [S]^{-1}\{P\}$

compatibility

Post-process $\{F\} = [k]\{v\}$

$\{Q\} = [T]\{F\}$ $\quad * \{F\} = [T]^T\{Q\}$

Alternatively: $\{v\}$ from $\{d\}$ $\quad \{u\} = [T]\{v\}$ $\quad \{Q\} = [k]\{u\}$