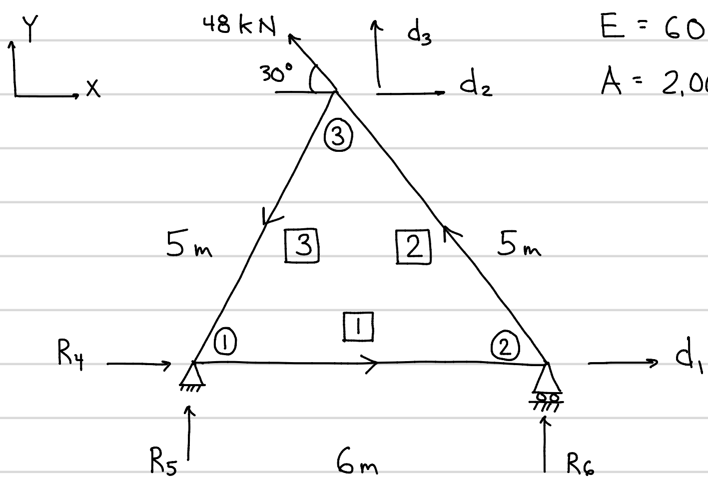
March 05, 2026
Matrix Displacement Method
2D Trusses
Part III - Numerical Example



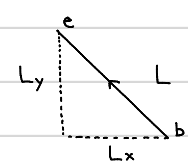
$E = 60\ GPa$ * $GPa = kN/mm^2$

$A = 2{,}000\ mm^2$

1. Label Structure
   - joints
   - members (local axes)
   - DOFs
   - reactions

2. Create force vector $\{P\}$

$$\{P\} = \begin{Bmatrix} 0 \\ P_x \\ P_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ -48\cos 30 \\ 48\sin 30 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -41.57 \\ 24 \end{Bmatrix} kN$$



$\cos = \frac{L_x}{L}$ $\sin = \frac{L_y}{L}$

$L_x \equiv X_e - X_b$ (-) $L_y \equiv Y_e - Y_b$ (+)

3. Determine local $[k]$ and transformation $[T]$

$$[k] = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad [T] = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

1 $\frac{EA^1}{L} = \frac{(60)(2{,}000)}{6{,}000} = 20\ \frac{kN}{mm}$ $\quad c = \frac{L_x}{L} = \frac{6}{6} = 1 \quad s = \frac{L_y}{L} = \frac{0}{6} = 0 \quad [T] = [I]$

2 $\frac{EA^2}{L} = \frac{(60)(2{,}000)}{5{,}000} = 24\ \frac{kN}{mm}$ $\quad c = \frac{-3}{5} \quad s = \frac{4}{5}$

3 $\frac{EA^3}{L} = 24\ \frac{kN}{mm}$ $\quad c = \frac{-3}{5} \quad s = \frac{-4}{5}$

Compute global $[k] = [T]^T [k] [T]$

$$[k]^1 = \begin{bmatrix} 20 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 \\ -20 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{kN}{mm}$$

$$[k]^2 = \begin{bmatrix} 8.64 & -11.52 & -8.64 & 11.52 \\ -11.52 & 15.36 & 11.52 & -15.36 \\ -8.64 & 11.52 & 8.64 & -11.52 \\ 11.52 & -15.36 & -11.52 & 15.36 \end{bmatrix} \frac{kN}{mm}$$

$$[k]^3 = \begin{bmatrix} 8.64 & 11.52 & -8.64 & -11.52 \\ 11.52 & 15.36 & -11.52 & -15.36 \\ -8.64 & -11.52 & 8.64 & 11.52 \\ -11.52 & -15.36 & 11.52 & 15.36 \end{bmatrix} \frac{kN}{mm}$$

4. Assemble $[S]$ using Code # Method

$$[S] = \begin{bmatrix} K_{33}^1 + K_{11}^2 & K_{13}^2 & K_{14}^2 \\ K_{31}^2 & K_{33}^2 + K_{11}^3 & K_{34}^2 + K_{12}^3 \\ K_{41}^2 & K_{43}^2 + K_{21}^3 & K_{44}^2 + K_{22}^3 \end{bmatrix}$$

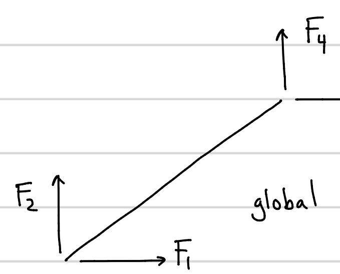
5. Solve $\{P\} = [S]\{d\}$ $\quad \{d\} = [S]^{-1}\{P\}$ $\quad \{d\} = \begin{Bmatrix} -1.49 \\ -3.15 \\ 1.34 \end{Bmatrix}$ mm

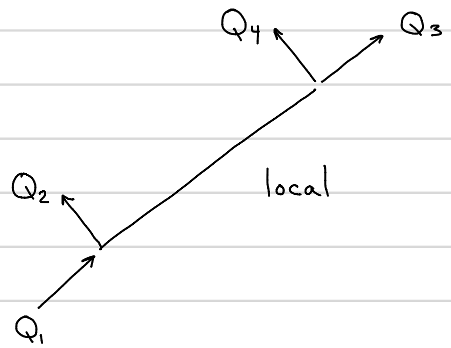
6. Compute global member end forces

$$\{F\} = [K]\{v\}$$ use compatibility to determine $\{v\}$ from $\{d\}$ and B.C.s

$$\{v\}^1 = \begin{Bmatrix} 0 \\ 0 \\ d_1 \\ 0 \end{Bmatrix} \begin{matrix} \text{pin} \\ \text{pin} \\ \\ \text{roller} \end{matrix} \qquad \{v\}^2 = \begin{Bmatrix} d_1 \\ 0 \\ d_2 \\ d_3 \end{Bmatrix} \begin{matrix} \\ \text{roller} \\ \\ \\ \end{matrix} \qquad \{v\}^3 = \begin{Bmatrix} d_2 \\ d_3 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} \\ \\ \text{pin} \\ \text{pin} \end{matrix}$$

$$\{F\}^1 = \begin{Bmatrix} 29.785 \\ 0 \\ -29.785 \\ 0 \end{Bmatrix} \text{kN} \qquad \{F\}^2 = \begin{Bmatrix} 29.785 \\ -39.713 \\ -29.785 \\ 39.713 \end{Bmatrix} \qquad \{F\}^3 = \begin{Bmatrix} -11.785 \\ -15.713 \\ 11.785 \\ 15.713 \end{Bmatrix}$$





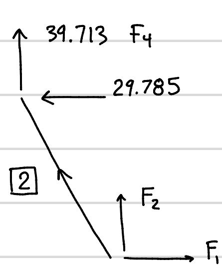
$Q_2 = Q_4 = 0$ (truss)

Q_3 (+) tension
(−) compression

7. Calculate axial bar forces/stresses

Opt. 1 Compute $\{Q\} = [T]\{F\}$ $\quad$ * $\{Q\} = [k]\{u\}$ $\quad$ $\{u\} = [T]\{v\}$

Opt. 2 Pythagorean Theorem



$$\sqrt{29.785^2 + 39.713^2} = 49.64 \text{ kN (T)}$$

To determine tension/compression need to evaluate respective vector components

Normal stress $\sigma_a = \frac{Q_3}{A}$ Compare applied stress to ultimate strength (σ_u)

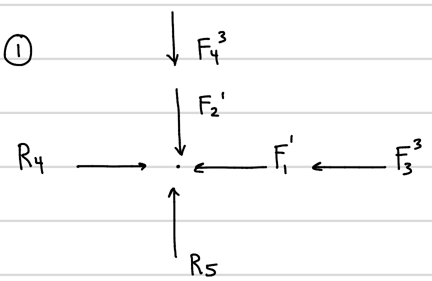
$|\sigma_a| \leq \sigma_u$ okay!

* For compression also check stability : $P_{cr} = \frac{\pi^2 EI}{(kL)^2}$ k - depends on B.C.s

$|Q_3| \leq P_{cr}$ okay! pin-pin $k = 1.0$

8. Determine support reactions from joint equilibrium (global F)

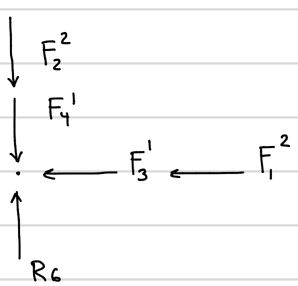
joint ①



$\sum F_x = 0 \quad R_4 = F_1^1 + F_3^3 = 29.785 + 11.785 = 41.57$ kN

$\sum F_y = 0 \quad R_5 = F_2^1 + F_4^3 = \emptyset + 15.713 = 15.713$ kN

joint ②



$\sum F_y = 0 \quad R_6 = F_4^1 + F_2^2 = \emptyset - 39.713 = -39.713$ kN

* Can check overall equilibrium of structure to verify correct solution