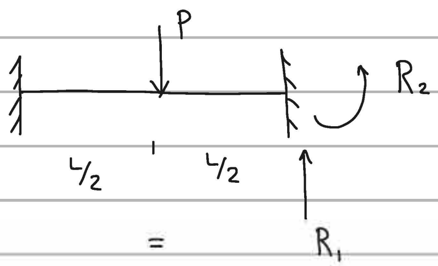
March 24, 2026
Matrix Displacement Method
Beams
Part II - Qf

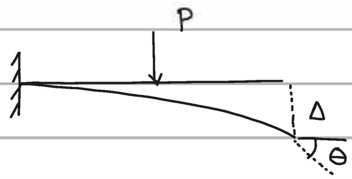
Fixed-end Forces/Moments

Indeterminate beam, use superposition



Identify redundant reactions (2)

=

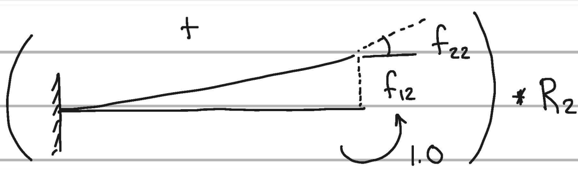


Apply loading, $R_1 = R_2 = 0$

+

$R_1 = 1.0, \; R_2 = 0$

+



$R_1 = 0, \; R_2 = 1.0$

Compatibility

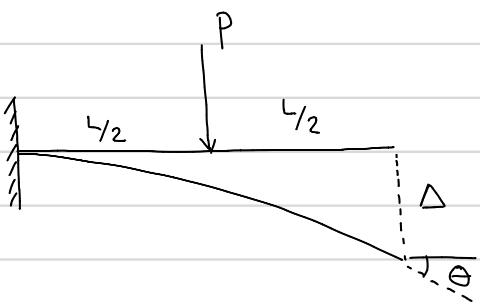
$$0 = \Delta + f_{11} R_1 + f_{12} R_2$$

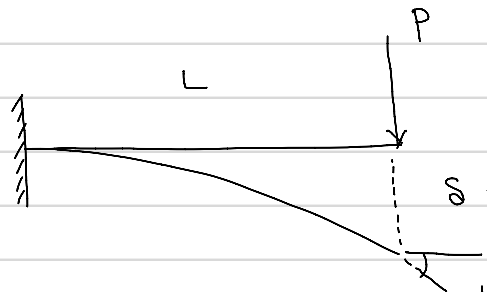
$$0 = \theta + f_{21} R_1 + f_{22} R_2$$

* Recall $\quad f_{11} = \dfrac{L^3}{3EI} \qquad f_{12} = \dfrac{L^2}{2EI}$

$\qquad f_{21} = \dfrac{L^2}{2EI} \qquad f_{22} = \dfrac{L}{EI}$

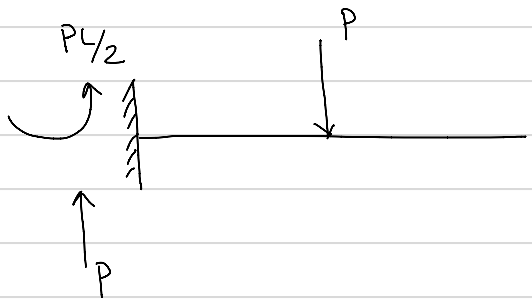
If we determine Δ, θ then can solve for redundant reactions R_1 and R_2

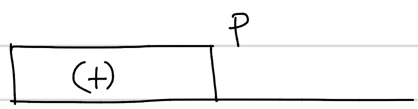




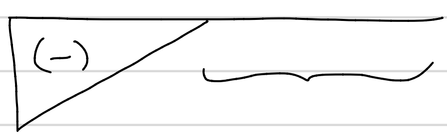
$$\delta = \frac{-PL^3}{3EI}$$

$$\phi = \frac{-PL^2}{2EI}$$





$V(x)$



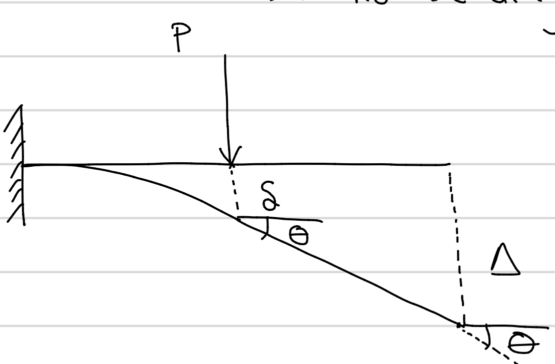
$M(x)$

$$EI\,v'' = M(x)$$

moment-curvature

curvature = 0

∴ no bending, straight segment ∴ rotation constant



$$\theta = -\frac{P(L/2)^2}{2EI} = \frac{-PL^2}{8EI}$$

$$\Delta = -\frac{P(L/2)^3}{3EI} + \theta \cdot L/2 = \frac{-5PL^3}{48EI}$$

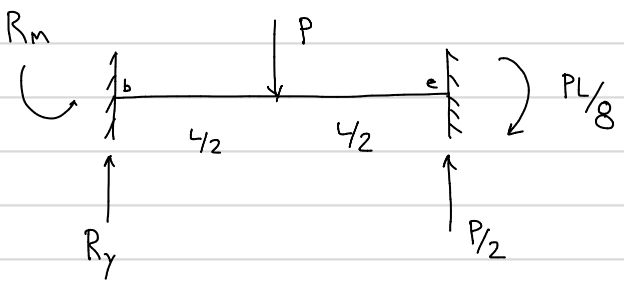
$$0 = \frac{-5PL^3}{48EI} + \frac{R_1 L^3}{3EI} + \frac{R_2 L^2}{2EI}$$ displacement compatibility

$$0 = \frac{-PL^2}{8EI} + \frac{R_1 L^2}{2EI} + \frac{R_2 L}{EI}$$ rotation compatibility

$$\begin{Bmatrix} \frac{5PL^3}{48EI} \\ \frac{PL^2}{8EI} \end{Bmatrix} = \begin{bmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix}$$

Solving system of equations $R_1 = P/2$ fixed-end-force (FEF)

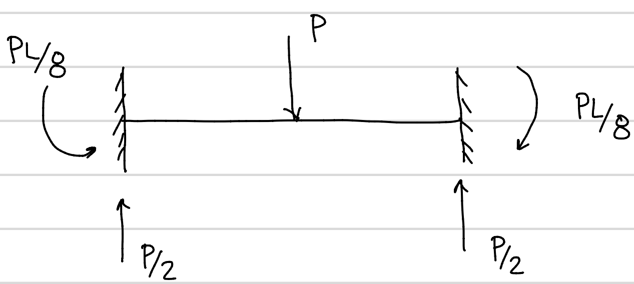
$R_2 = -PL/8$ fixed-end-moment (FEM)



From static equilibrium (determinate now)

$\sum F_y = 0 \quad R_y + P/2 - P = 0$

$\sum M_b = 0 \quad R_m - P(L/2) + P/2(L) - PL/8 = 0$



$$\{Qf\} = \begin{Bmatrix} Qf_1 \\ Qf_2 \\ Qf_3 \\ Qf_4 \end{Bmatrix} = \begin{Bmatrix} P/2 \\ PL/8 \\ P/2 \\ -PL/8 \end{Bmatrix}$$ ⟵ use correct sign!